GRAPHS and FUNCTIONS

CIRCLES

Terrametra Resources

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2.2
CIRCLES

- Center-Radius Form
- General Form
- An Application

CIRCLES

CIRCLES

A ***circle*** is the set of all points in a plane that lie a given distance from a given point.

The given distance is the ***radius*** of the circle, and the given point is the ***center***.

Center-Radius Form of the Equation of a Circle

Center-Radius Form of the Equation of a Circle

A circle with center $(\boldsymbol{h}, \boldsymbol{k})$ and radius $\boldsymbol{r}$ has equation …

$$(\boldsymbol{x}-\boldsymbol{h})^{2}+(\boldsymbol{y}-\boldsymbol{k})^{2}=\boldsymbol{r}^{2}$$

which is the ***center-radius form*** of the equation of a circle.

A circle with center $(\mathbf{0}, \mathbf{0})$ and radius $\boldsymbol{r}$ has equation …

$$\boldsymbol{x}^{2}+\boldsymbol{y}^{2}=\boldsymbol{r}^{2}$$

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Example 1

Finding the Center-Radius Form

1(a) Find the center-radius form of the equation of a circle with center $(-3, 4)$ and radius 6.

Solution:

$$(x-h)^2+(y-k)^2=r^2$$ Center-radius form.

$$[x-(-3)]^2+(y-4)^2=6^2$$ Substitute.

Let $(h, k) = (-3, 4)$ and $r = 6$.

Watch signs here.

$$(x+3)^2+(y-4)^2=36$$ Simplify.

Example 1

Finding the Center-Radius Form

1(b) Find the center-radius form of the equation of a circle with center $(0, 0)$ and radius 3.

Solution:

The center is the origin and $r = 3$.

$$x^2 + y^2 = r^2$$ Center-radius form.

$$x^2 + y^2 = 3^2$$ Substitute.

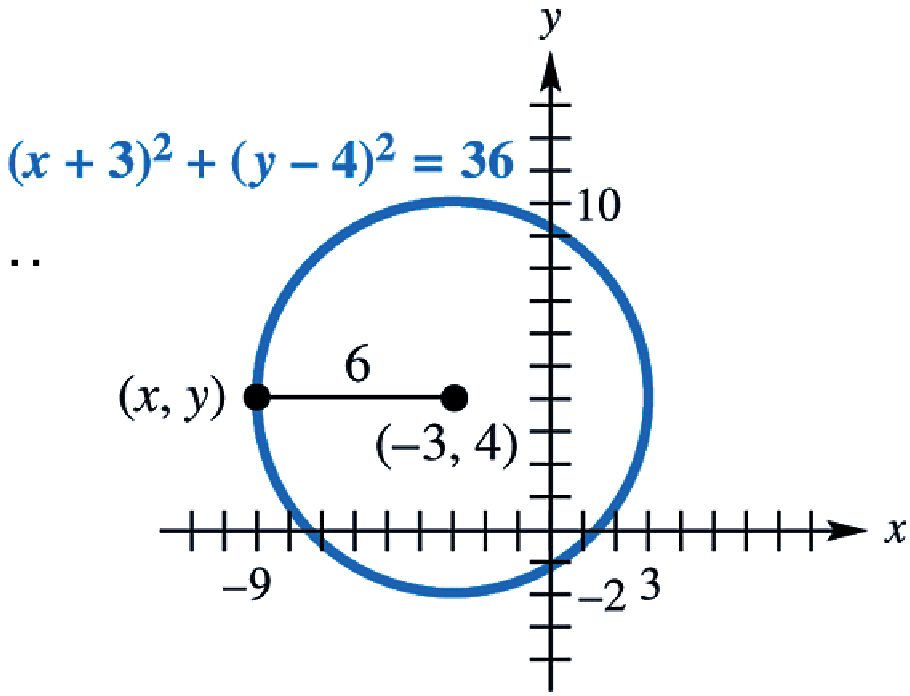
$$x^2 + y^2 = 9$$ Simplify.

Example 2
Graphing Circles

2(a) Graph the circle discussed in **Example 1a**.

$$(x+3)^2+(y-4)^2=36$$

Solution:



Writing in center-radius form …

$$[x-(-3)]^2+(y-4)^2=6^2$$

… gives $(-3,4)$ as the center and 6 as the radius.

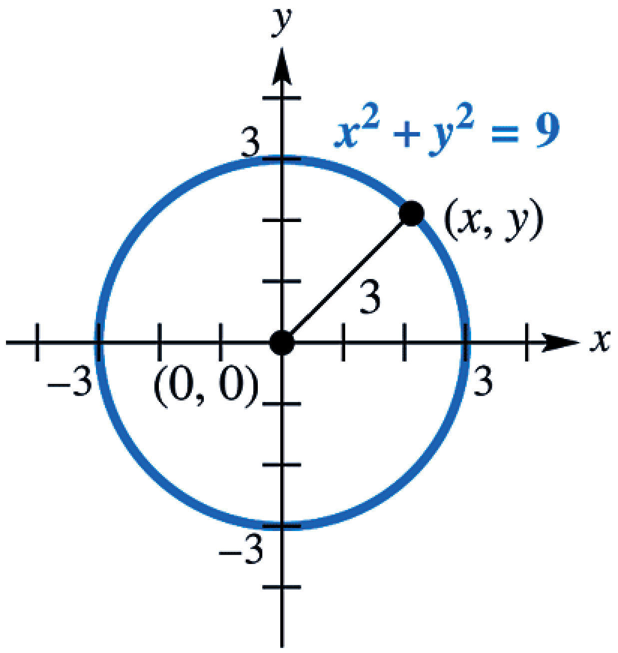
Example 2

Graphing Circles

2(b) Graph the circle discussed in **Example 1b**.

$$x^2 + y^2 = 9$$

Solution:



Writing in center-radius form …

$$x^2 + y^2 = 3^2$$

… gives $(0, 0)$ as the center and 3 as the radius.

General Form of the Equation of a Circle

General Form of the Equation of a Circle

For some real numbers D, E, and F, the equation …

$$\boldsymbol{x^2 + y^2 + Dx + Ey + F = 0}$$

… can have a graph that is a circle or a point or be nonexistent.

General Form of the Equation of a Circle

Consider $(\boldsymbol{x}-\boldsymbol{h})^2+(\boldsymbol{y}-\boldsymbol{k})^2=\boldsymbol{c}$

There are three possibilities for the graph based on the value of $\boldsymbol{c}$ …

1. If $\boldsymbol{c}>\mathbf{0}$, then $\boldsymbol{r}^2=\boldsymbol{c}$,
   and the graph of the equation is a circle with radius $\sqrt{\boldsymbol{c}}$.
2. If $\boldsymbol{c}=\mathbf{0}$, then the graph of the equation is the single point $(\boldsymbol{h},\boldsymbol{k})$.
3. If $\boldsymbol{c}<\mathbf{0}$, then no points satisfy the equation
   and the graph is nonexistent.

Example 3

Finding the Center and Radius by Completing the Square

3(a) Show that $x^2 - 6x + y^2 + 10y + 18 = 0$ …
has a circle as its graph. Find the center and radius.

Solution:

Complete the square twice, once for x and once for y.

$$x^2 - 6x + y^2 + 10y + 18 = 0$$

$$(x^2 - 6x \quad\quad) + (y^2 + 10y \quad\quad) = -18$$

$$\left[\frac{1}{2}(-6)\right]^2 = (-3)^2 = 9 \qquad\qquad \left[\frac{1}{2}(10)\right]^2 = 5^2 = 25$$

Add 9 and 25 on the left to complete the two squares,
and to compensate, add 9 and 25 on the right.

Example 3

Finding the Center and Radius by Completing the Square

3(a) Show that $x^2 - 6x + y^2 + 10y + 18 = 0$ …
has a circle as its graph. Find the center and radius.

Solution (cont'd):

Add 9 and 25 on *both* sides.

Complete the square.

$$(x^2 - 6x + 9) + (y^2 + 10y + 25) = -18 + 9 + 25$$

$$(x-3)^2 + (y+5)^2 = 16$$ Factor.

$$(x-3)^2 + [y-(-5)]^2 = 4^2$$

Because $4^2 = 16$ and $16 > 0$, the equation represents a circle with center $(3, -5)$ and radius 4.

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Example 4

Finding the Center and Radius by Completing the Square

4(a) Show that $2x^2 + 2y^2 - 6x + 10y = 1$ …
has a circle as its graph. Find the center and radius.

Solution:

To complete the square, the coefficients of the x^2- and y^2-terms must be 1.

$$2x^2 + 2y^2 - 6x + 10y = 1$$

$$x^2 + y^2 - 3x + 5y = \frac{1}{2}$$

Divide by 2.

$$(x^2 - 3x \quad) + (y^2 + 5y \quad) = \frac{1}{2}$$

Rearrange and regroup terms.

Example 4

Finding the Center and Radius by Completing the Square

4(a) Show that $2x^2 + 2y^2 - 6x + 10y = 1$ … has a circle as its graph. Find the center and radius.

Solution (cont'd): Complete the square for <u>both</u> x and y.

$$\left(x^2 - 3x + \frac{9}{4}\right) + \left(y^2 + 5y + \frac{25}{4}\right) = \frac{1}{2} + \frac{9}{4} + \frac{25}{4}$$

$$\left(x - \frac{3}{2}\right)^2 + \left(y + \frac{5}{2}\right)^2 = 9$$

Factor and add.

$$\left(x - \frac{3}{2}\right)^2 + \left(y - \left(-\frac{5}{2}\right)\right)^2 = 3^2$$

Center-radius form.

The equation has a circle as its graph with center at $\left(\frac{3}{2}, -\frac{5}{2}\right)$ and radius 3.

Example 5

Determining Whether a Graph is a Point or Nonexistent

5(a) The graph of $x^2 + 10x + y^2 - 4y + 33 = 0$ … is either a point or is nonexistent. Which is it?

Solution:

$$x^2 + 10x + y^2 - 4y + 33 = 0$$

$$x^2 + 10x + y^2 - 4y = -33$$ Subtract 33.

$$\left[\frac{1}{2}(10)\right]^2 = 25 \qquad \left[\frac{1}{2}(-4)\right]^2 = 4$$

$$(x^2 + 10x + 25) + (y^2 - 4y + 4) = -33 + 25 + 4$$

Complete the square.

Example 5

Determining Whether a Graph is a Point or Nonexistent

5(a) The graph of $x^2 + 10x + y^2 - 4y + 33 = 0$ … is either a point or is nonexistent. Which is it?

Solution (cont'd):

$$(x^2 + 10x + 25) + (y^2 - 4y + 4) = -33 + 25 + 4$$

$$(x+5)^2 + (y-2)^2 = -4$$ Factor; add.

Since $-4 < 0$, there are no ordered pairs (x, y), with x and y both real numbers, satisfying the equation …

The graph of the given equation is nonexistent.

If the constant on the right side were 0, the graph would consist of the single point $(-5, 2)$.

Example 6

Determining the Epicenter of an Earthquake

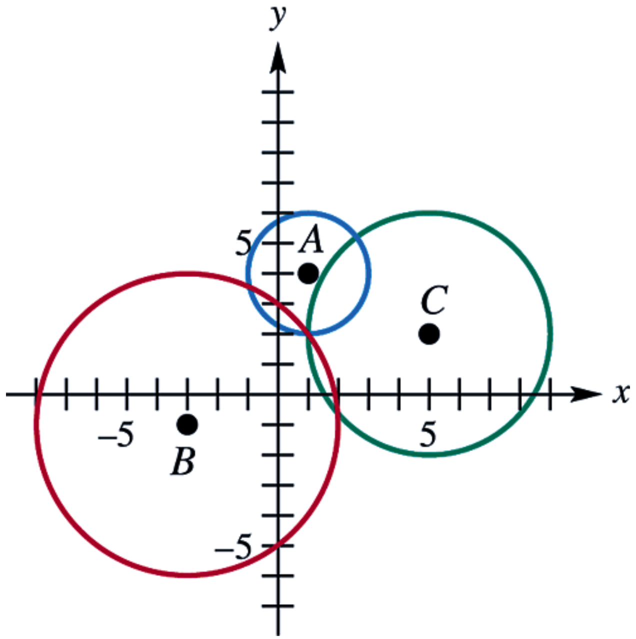
Suppose receiving stations *A*, *B*, and *C* are located on a coordinate plane at the points $(1, 4)$, $(-3, -1)$, and $(5, 2)$. Let the distances from the earthquake epicenter to these stations be 2 units, 5 units, and 4 units, respectively. Where on the coordinate plane is the epicenter located?

Solution:

Graph the three circles.

From the graph it appears that the epicenter is located at $(1, 2)$.

To check this algebraically, determine the equation for each circle and substitute $x = 1$ and $y = 2$.



Example 6

Determining the Epicenter of an Earthquake

Station *A*:

$$(x-1)^2+(y-4)^2=4$$
$$(1-1)^2+(2-4)^2\stackrel{?}{=}4$$
$$0+4\stackrel{?}{=}4$$
$$4=4$$

Station *B*:

$$(x+3)^2+(y+1)^2=25$$
$$(1+3)^2+(2+1)^2\stackrel{?}{=}25$$
$$16+9\stackrel{?}{=}25$$
$$25=25$$

Station *C*:

$$(x-5)^2+(y-2)^2=16$$
$$(1-5)^2+(2-2)^2\stackrel{?}{=}16$$
$$16+0\stackrel{?}{=}16$$
$$16=16$$

Since the algebraic check results in true statements for all three circles, the point $(1, 2)$ is the epicenter.